Directionality of sperm whale sonar clicks and its relation to piston radiation theory

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Abstract: This paper investigates the applicability to sperm whales of the theory of sound radiating from a piston. The theory is applied to a physical model and to a series of sperm whale clicks. Results show that wave forms of off-axis signals can be reproduced by convolving an on-axis signal with the spatial impulse response of a piston. The angle of a recorded click can be estimated as the angle producing the spatial impulse response that gives the best match with the observation when convolved with the on-axis wave form. It is concluded that piston theory applies to sperm whale sonar click emission.

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1. Introduction

Odontocete whales use directional clicks for echolocation.[1] The clicks are generated in the nasal complex below the blowhole and transmitted via the melon to the water.[2] In dolphins where this has been investigated, it has been shown that the directional pattern of the beam can be modeled to a first approximation if one assumes the sound generator to be a flat circular piston.[3] The question is whether this model can be applied to the sperm whale that has a radically hypertrophied sound-generating nasal complex.[2]

In recordings of free-ranging sperm whales, there is no simple way to determine the relative orientation of the animal with respect to the hydrophone. The recording aspect to a piston transducer not only influences the recorded peak amplitude of the transmitted signal relative to the on-axis signal, but it also affects the wave form in a predictable way.[4] If the piston model is in fact applicable to sperm whales, it should be possible to determine the angle to the acoustic axis of the animal by comparing the observed wave forms in a scan of clicks with theoretical ones obtained from two parameters, namely the diameter of the piston and the wave form of the on-axis signal. The observed amplitudes can then be plotted against the estimated angles and compared with the predictions from the piston model.

To address the applicability of this method, we evaluated the response of an experimental physical piston, namely an electrostatic loudspeaker,[5] and compared the results to a sequence of sperm whale clicks (Fig. 1). The speaker is known to behave roughly like a piston, but has some features in common with the sperm whales, such as the lack of a baffle and presumably also some effects of the edge being less active than the center. We therefore assume that if the proposed method performed satisfactorily with the speaker, it would be likely to work with the sperm whales too.

2. Materials and methods

The theory describing the directionality of a piston transducer is well established. In textbooks, one finds that the radiation pattern of a circular baffled piston transmitting a given sound can be

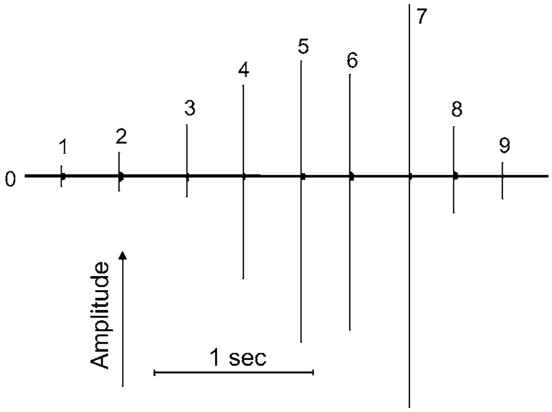

Fig. 1. Example of a scan. Sperm whale clicks from a single animal (48 kHz sampling rate) recorded at Andenes, Norway (see Ref. 4).

computed by modifying the complex spectrum of the sound by multiplication with a so-called Jinc function.[7] This spectral approach is cumbersome to derive and it is demanding to visualize the physical reasoning behind it. In contrast, it is quite straightforward to grasp the angle-dependent wave form of a piston transmitting an impulse in the time domain. In agreement with Harris[8] (citing Rutgers),[9] we shall denote this the "spatial impulse response." The appearance of any transmitted sound at a given observation angle is found by convolving the angle-dependent spatial impulse response of the piston with the sound that was observed on axis.

The two approaches, the multiplication of the complex spectrum with a Jinc function and the convolution in the time domain with the corresponding spatial impulse response, give exactly the same results. Here, we use the time domain representation, which makes it simpler to comprehend the geometry of the process.

In the direction perpendicular to the surface of a circular piston, the spatial impulse response is a theoretical Dirac function, since (per definition) it does not alter the transmitted sound through convolution. The impulse response obtained at any other angle takes the shape of a half-ellipse when graphed. To recognize this, one must accept that the pulse projected from different parts of the piston surface arrives at the receiver with different delays: The first part to arrive was projected by the closest edge, the last part of the received signal by the edge farthest away. The delay between these two time instances is (see Fig. 2)

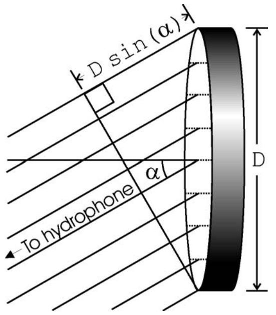

Fig. 2. Geometry of a piston transducer radiating sound as seen at a long distance relative to the diameter of the surface, illustrating the reasoning behind the spatial impulse response of a circular piston. All points on the surface radiate as a point source. If the radiated sound is an infinitely short, positive-going pulse, then the duration of the received sound becomes $\sin(\alpha)D/c$. During this time, the amplitude of the recorded sound is given by the length of the cross section (dotted lines on the surface of the disk) that projects the pulse at any one moment, leading to the sound resembling an ellipse when graphed.

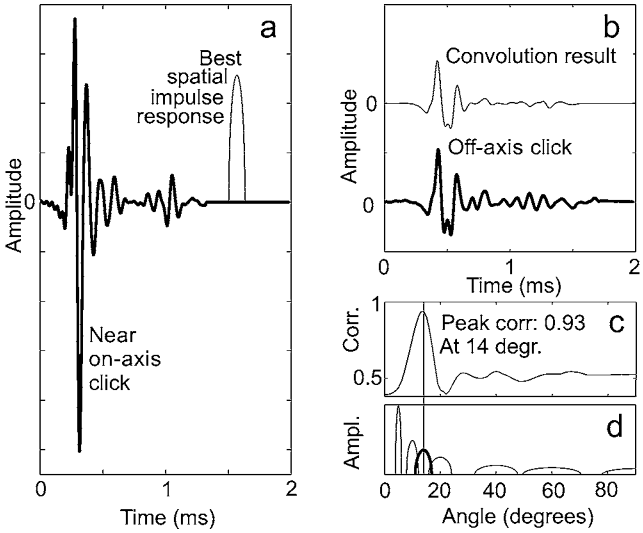

Fig. 3. Convolution and matching method for a sperm whale click. The two clicks in this example are from a scan other than the one shown in Fig. 1. (a) *Thick line* is a click judged to be recorded on axis. *Thin line* is the spatial impulse response of a piston at 14° (assuming $\varnothing = 0.8$ m). (b) *Thin line* is the convolution of the two wave forms in (a). *Thick line* is the observed off-axis click to be matched. Vertical scale is the same as in (a). (c) Correlation values between the observed click in (b) and spatial impulse responses of a piston at different angles. (d) Spatial impulse response of a piston ($\varnothing = 0.8$ m) in salt water at various angles. Time axis for these spatial impulse responses is the same as in (a) and (b). The spatial impulse response shown with *thick line* is the same as the thin line in (a).

$$T = \sin(\alpha)\frac{D}{c}. \quad (1)$$

During this time interval, the instantaneous cross section of the piston to transmit the sound is described as the edge-to-edge distances in a circle, measured perpendicularly to, e.g., its cosine axis. This is then the sine of the angle corresponding to the cosine value. After normalization, the resulting expression equals:

$$p(\alpha,t) = \frac{4}{\pi T}\sin\left(\cos^{-1}\left(\frac{2t}{T}\right)\right), \quad |t| < T/2, \quad 0 \text{ elsewhere}. \quad (2)$$

The factor $4/(\pi T)$ ensures that the integral is unity. Examples of such functions for different angles are given in Fig. 3(d). The expression (2) belongs to the class of functions that converge on a theoretical Dirac function as the duration goes to zero.

For both the loudspeaker and the sperm whale, $p(\alpha,t)$ was calculated for a series of closely spaced angles to make up an array of vectors representing different receiving angles. The on-axis wave form was convolved with each of these spatial impulse responses. Then, to determine the recording angle, α, of each of the (assumed) off-axis sounds, the best match was picked from the array of on-axis sounds convolved with the spatial impulse responses, $p(\alpha,t)$.

The physical model experiment was made in air with an electrostatic loudspeaker ($\varnothing = 6$ cm) projecting clicks with a peak frequency of 40 kHz. The speaker was mounted on a machinist's dividing head with an angular Vernier scale. The clicks were recorded with a 0.25 in. B&K microphone (Model 4135) in a series of angles relative to the normal to the membrane. The distance[10] to the microphone was 1 m. At each angle, 256 signals were averaged after sampling with 8-bit resolution at 5 MHz.

The sperm whale click series in Fig. 1 was selected for analysis. It was recorded (16 -bit resolution, 48 kHz sampling rate)[6] at Bleijk Djup, Andenes, Norway, July 21, 2000. The whale was at a range of 1.1 km from the hydrophone and a depth of 0.64 km. The hydrophone

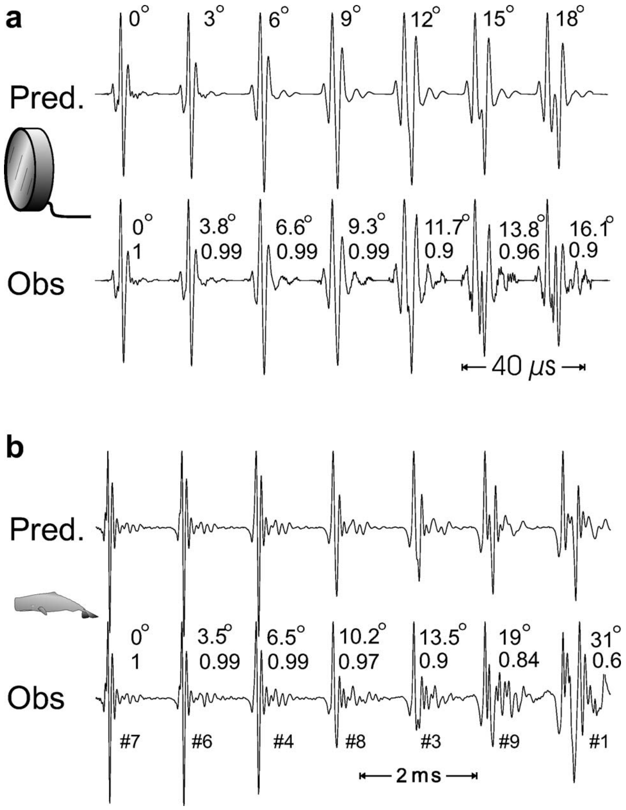

Fig. 4. Predicted and observed wave forms for an electrostatic speaker and a sperm whale. (a) Normalized click wave forms predicted and recorded from a electrostatic speaker (∅=6 cm) transmitting at the angles given above the upper traces in air. The predictions were made by convolving the wave form recorded at 0° with the relevant piston spatial impulse response. The correlation values are shown above the "observed" traces, below the angle estimated by means of the proposed method of searching for the best match between a piston transmitting the on-axis signal at different angles and the observed signal. (b) Comparison between sperm whale clicks (selected from Fig. 1) and the best matching results of submitting Click No. 7 in Fig. 1 to the piston model at different angles (with an assumed diameter of 0.8 m—see Ref. 10). The best matching angle is shown above the correlation values. Numbers below the "observed" traces indicate the pulse number in Fig. 1.

was at a depth of 30 m. From the interpulse interval of 7.4 ms, it was found[11] that the length of the animal was 16 m and, by inference, a male. Click No. 7 in Fig. 1 was considered to be on axis or nearly so.[6] It is, however, important to recognize that the degree to which a click is on axis cannot be quantified. It is further assumed that all the other clicks in the series were identical in wave form and would have had the same amplitude, had they also been recorded on axis. Since the clicks are not similar, it is conjectured that the whale is scanning with a narrow beam of sound across the general area where the hydrophone is deployed so that all clicks other than No. 7 are recorded at various angles off axis. For the theoretical predictions, the radiating aperture was tentatively set to have a cross section[12] of 0.8 m. With these assumptions, we searched for the highest correlation value between the actual observed off-axis clicks and the spatial impulse response of a piston transmitting the on-axis click [Fig. 4(b)] at 0°–90° angles (see Fig. 3) as described above. A radiation diagram was constructed for the sperm whale by plotting the peak amplitudes against the angle estimate.

3. Results and discussions

The results of the model experiment are shown in Fig. 4(a). Here, the estimated angles and the theoretically calculated wave forms are shown in the upper trace, and the known angles and the

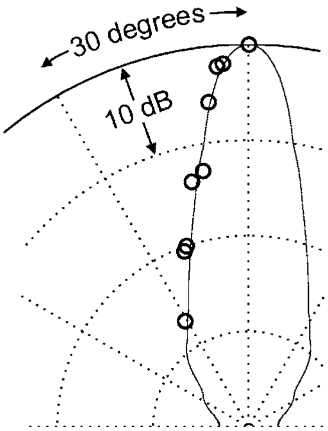

Fig. 5. Predicted and observed radiation patterns. *Thick line*: Radiation diagram of Click No. 7 in Fig. 1 (template) in a piston model with ∅=0.8 m. *Circles*: Amplitudes of the clicks in Fig. 1 plotted against the angle at which they match best with Click No. 7 as the template that was submitted to the piston model.

actually observed wave forms are shown below. The correlation between predicted and observed wave forms is generally high, highest for the smallest angles. The accuracy of the angle estimates was within 2°. This confirms that for a system known to conform to a piston model, it is possible to predict the wave form as seen off axis, using the on-axis wave form and the method proposed above. This result serves as an indicator of the way we can expect the sperm whale clicks to behave if indeed the piston model also applies to the animals. There seems to be a trend to the slight errors in the estimation of the angle, which may reflect the fact that there is no baffle and a slightly raised edge on the loudspeaker.

Figure 4(b) shows the observed and predicted wave forms for the sperm whale echolocation clicks of Fig. 1. The correlation values between them are above 0.9 out to 13.5: It therefore does appear that the observed sperm whale click wave forms are predictable as a convolution product between the on-axis click and a piston acoustic profile.

The estimated angles are on the low side in the model experiment. It is likely that this underestimation of the actual angle of recording in the case of the speaker at angles of 15° and 18° was caused by either imperfections in the behavior of the membrane or some edge effect. For the same reasons, we might suspect that the angles calculated for the sperm whales using this method will generally tend to be underestimated above, say, 12°.

Considering these caveats, the radiation diagram in Fig. 5, made by plotting the observed amplitudes of the clicks against the angle estimated with the method described in Fig. 3, does suggest that the piston model is quite applicable to these sperm whale sonar clicks, at least at these modest angles. The function in Fig. 5 (solid line) is a radiation diagram constructed from one on-axis click (Fig. 1, Click No. 7). The points are plotted at the angle of best fit for the individual click in the scan against its amplitude as obtained from Fig. 1. The degree to which the amplitude values lie on the predicted radiation diagram is independent of the computation of angle: If the sound, assumed to be recorded on axis, was in fact not so, the emerging pattern should not fit the model, and more importantly, if the model is not applicable, the beam pattern computed in this way will not comply with the prediction. The diameter of the piston does not influence how well the theoretical curve aligns with the individual amplitude points; changing the diameter changes the angles of both by an equal amount.

One interesting alternative approach to the angle estimation procedure used here is to let the loudspeaker project a scaled sperm whale click and look for the angle to the speaker where the wave form best matches the off-axis sperm whale click, in essence skipping the theoretical step. For this approach to work, however, one must deconvolve the suitably timecompressed sperm whale click by the impulse response of the loudspeaker. In our setup, this was simply too noisy to be feasible. Several possibilities for future applications arise from the observation that the piston model seems to explain the field data.

Source levels of odontocete clicks are a much wanted datum for biosonar analysis, yet difficult to obtain in the field due to directionality and unknown orientation of the animals. While in theory it should be possible to reverse the processing outlined in Fig. 3 to obtain the on-axis signal from an off-axis representation, such an approach is confounded by zeroes in the Fourier transform of the convolution functions. However, an estimate of the on-axis amplitude may be obtained from an off-axis click by plotting the relevant piston radiation function and read out the difference in level between the estimated angle and the on-axis direction, and add this to the received level of the off-axis click.

Another promising prospect lies in collecting a larger number of sounds fulfilling the on-axis criteria.[6] It might then be possible to derive set of common characteristics (the generic sperm whale click) that allows for the determination of the off-axis angle at which any sperm whale click encountered is most likely to have been observed. With actual measured values of the angle to a sperm whale, it is possible to calibrate the success of such a general template in producing the correct angles.

Such future applications require that the diameter of the equivalent piston be determined, e.g., by recording the same click at several angles, including 0°, simultaneously. The size of the animal can be determined by applying the Gordon equation[11] to the interpulse interval of the clicks in the scan as it was done above. The piston diameter for a sperm whale of a given size can then be obtained by rearranging Eqs. (1) and (2)

Other odontocetes may be modeled in a similar way making it possible to estimate the recording angle from the characteristics of recorded clicks.

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